The Diffeomorphism Group of Manifolds

A Hybrid Reading Seminar

Fall Semester 2025

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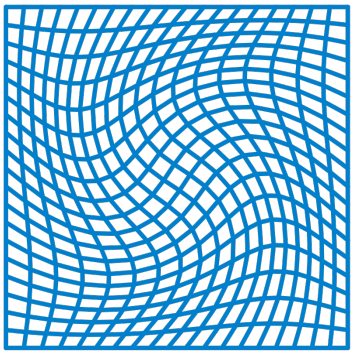

The image of a rectangular grid on a square under a diffeomorphism.
Picture by Oleg Alexandrov

Description

In life as well as in mathematics, it is not important if you come back where you started, but it is the journey you followed that makes you who you are. That is why this seminar is dedicated to the study of the Diffeomorphism Group $G = \{F \colon M \to M \mid F \text{ is a diffeomorphism}\}$ of a manifold M.

We will learn how the algebraic structure of G is intimately related to geometry and dynamics on M, and how the phenomena we observe change in the topological, differentiable and symplectic category. In particular, we show that G is (in most cases) simple, that is, it has no non-trivial normal subgroups. As a corollary, we prove that two groups are isomorphic if and only if the manifolds are isomorphic. In the process, we meet useful tools such as Haefliger structures, fragmentation of diffeomophisms, and the flux homomorphism. Our main source is Banyaga, *The Structure of Classical Diffeomorphism Groups* [2].

If time permits, we can delve into a series of spin-offs such as the Hilbert–Smith conjecture about faithful actions of locally compact groups, and the Smale conjecture about the centralizer of a generic diffeomorphism.

Time and Space

The Seminar takes place every Tuesday at 11:00 at the Vrije Universiteit Amsterdam, NU-Building, Floor 9, Room NU-9A46.

It is possible to follow the seminar also via Zoom at
`vu-live.zoom.us/j/99134747510?pwd=5by9w5TVDWVlnLEkXxaAcdwPRfPw4j.1`

Subscribe and/or give a talk

Contact Gabriele at g.benedetti@vu.nl if you want to subscribe to the mailing list and/or give a talk. We can help you prepare and pair you with another person so that you can work together.

Plan

1. **(September 9)** Introduction to the seminar, forming the list of speakers
2. **(September 23)** Basics on diffeomorphism groups [2, Introduction, Chap. 1]
3. **(September 30)** From perfectness to simplicity [2, Chap. 2]
4. **(October 7)** The Epstein Theorem and Herman Theorem [2, Chap. 2]
5. **(October 14)** Foliations and equivariant diffeomorphism groups [2, Chap. 2], [6]
6. **(October 28)** The flux homomorphism [2, Chap. 3]
7. **(November 11)** Symplectic diffeomorphisms [2, Chap. 4]
8. **(November 25)** Volume-preserving diffeomorphisms [2, Chap. 5]
9. **(December 2)** Isomorphisms between diffeomorphism groups [2, Chap. 7]
10. **(February 10)** The simplicity of (volume-preserving) homeomorphisms [1, 5].
11. **(February 24)** The group of area-preserving homeomorphisms of the 2-disc is not simple [4]
12. **(March 10)** The Hilbert–Smith conjecture in the differentiable [8, 7] and symplectic category [9].
13. **(March 17)** Smale conjecture on centralizers of diffeomorphisms [3].

References

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[4] D. Cristofaro-Gardiner, V. Humilière, and S. Seyfaddini. Proof of the simplicity conjecture. *Ann. Math. (2)*, 199(1):181–257, 2024.

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[8] J. Pardon. Totally disconnected groups (not) acting on two-manifolds. In *Breadth in contemporary topology. 2017 Georgia international topology conference, University of Georgia, Athens, Georgia, USA, May 22 – June 2, 2017*, pages 187–193. Providence, RI: American Mathematical Society (AMS), 2019.

[9] E. Shelukin. A symplectic Hilbert–Smith conjecture. arXiv:2403.07195.