A new approach to Posterior Contraction Rates via Wasserstein dynamics

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joint work with

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Abstract

We presents a new approach to the problem of quantifying posterior contraction rates (PCRs) in Bayesian statistics. See [1]. Our approach relies on Wasserstein distance, and it leads to two main contributions which improve on the existing literature of PCRs. Both the results exploit a local Lipschitz-continuity of the posterior distribution on some sufficient statistic of the data (noteworthy, the empirical distribution). See also [2].

The first contribution involves the dynamic formulation of Wasserstein distance due to Benamou and Brenier—referred to as Wasserstein dynamics—in order to establish PCRs under dominated Bayesian statistical models. As a novelty with respect to existing approaches to PCRs, Wasserstein dynamics allows us to circumvent the use of sieves in both stating and proving PCRs, and it sets forth a natural connection between PCRs and three well-known problems in probability theory: the speed of mean Glivenko-Cantelli convergence, the estimation of weighted Poincaré-Wirtinger constants and Sanov large deviation principle for Wasserstein distance.

The second contribution combines the use of Wasserstein distance with a suitable sieve construction to establish PCRs under full Bayesian nonparametric models. As a novelty with respect to existing literature of PCRs, our second result provides with the first treatment of PCRs under non-dominated Bayesian models.

Applications of our results are presented for some classical Bayesian statistical models. By way of example, for the former result we discuss density estimation in a setting similar to that in [3], while for the latter we consider the Ferguson-Dirichlet process.

References

[1] Dolera, E., Favaro, S. and Mainini, E. (2020). A new approach to Posterior Contraction Rates via Wasserstein dynamics. *ArXiv:2011.14425*.

[2] Dolera, E. and Mainini, E. (2020). Lipschitz continuity of probability kernels in the optimal transport framework. *ArXiv:2010.08380*.

[3] Sriperumbudur, B., Fukumizu, K., Gretton, A., Hyvärinen, A. and Kumar, R. (2017). Density Estimation in Infinite Dimensional Exponential Families. *Journal of Machine Learning Research* **18**, 1-59.